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TRANSIENT ANALYSIS OF FATIGUE CRACK PROPAGATION UNDER A STEP INCREASE IN LOADING

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DEPARTMENT OF THEORETICAL AND APPLIED MECHANICS
UNIVERSITY OF ILLINOIS
URBANA, ILLINOIS

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TRANSIENT ANALYSIS OF FATIGUE CRACK PROPAGATION UNDER A STEP INCREASE IN LOADING

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Saurindranath Majumdar

This research was performed in the Department of Theoretical and Applied Mechanics at the University of Illinois at Urbana-Champaign, Illinois 61801 with support of the Advanced Research Projects Agency of the Department of Defense under Grant Nos. DAHC 15-72-G-10 and DAHC 15-73-G-7; ARPA Order No. 2169 w/Amend. 1, Req. No. 1001/191; Methods and Applications of Fracture Control. The period of the Grants is from June 15, 1972 through June 14, 1974 and the amount is $100,000/year. Professor H. T. Corten 217/333-3175 is Principal Investigator and Professors G. M. Sinclair 217/333-3173, JoDean Morrow 217/333-4167 and H. R. Jhansale 217/333-1835 have participated as Project Scientists.

Department of Theoretical and Applied Mechanics
University of Illinois
Urbana, Illinois
May, 1974

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FOREWORD

Recent requirements for increased strength and service life of machines and structures have been met by the use of higher strength materials and new fabrication and joining methods. Simultaneously, failures due to fracture have increased relative to those resulting from excessive deformation. Frequently service conditions are such that low temperature brittle fracture, fatigue fracture, and high temperature creep rupture must be considered in a single system. National concern with increased safety, reliability, and cost has focused attention upon these problems.

Methods are now available to predict both fatigue crack initiation life and crack propagation life. Paradoxically the materials properties required for long fatigue crack initiation life are incompatible with the requirements of high fracture toughness. Thus, the conflicting design approaches and requirements placed on the material are confusing and often impossible to satisfy.

Numerous publications dealing with a variety of fracture problems have led to many new and useful developments. However, the synthesis of the concepts into methods for design, testing and inspection has lagged.

This program of study is intended to contribute to the integration, correlation, and organization of mechanics and materials concepts and research information into a form that will permit enlightened decisions to be made regarding fracture control. Reports are in preparation in three categories:

1. Research reports designed to explore, study and integrate isolated and/or conflicting concepts and methods dealing with life prediction,
2. Reports to introduce and summarize the state-of-the-art concepts and methods in particular areas, and
3. Example problems and solutions intended to illustrate the use of these concepts in decision making.

H. T. Corten
Principal Investigator

SUMMARY

In a previous report, steady state crack propagation at a constant stress intensity factor range was analyzed on the basis of a cumulative fatigue damage model. In this model, the points ahead of a crack tip were assumed to constitute a series of uniaxial fatigue specimen and fatigue crack propagation was viewed as the successive fatigue failure of these specimens.

In this report an extension of the previous analysis to a situation involving an increase in the stress intensity factor range from a lower level to a higher level is carried out. The analysis indicates that there is a transient zone through which the crack has to grow before attaining the full steady state crack propagation rate corresponding to the higher load level. Numerical results have been computed for three steels of comparable ductility but with widely varying yield strength. The results show that both the crack length increment and the number of cycles involved in the transient period decrease with increasing yield strength. Also, for any given steel, the crack length increment during the transient increases with the load ratio but the number of cycles is independent of the load ratio.

ACKNOWLEDGMENT

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Professors JoDean Morrow and J. S. Walker contributed to this research through helpful discussion and critical review of the manuscript. Mrs. Darlene Mathine typed the manuscript.

INTRODUCTION

In a recent paper [1], steady state fatigue crack propagation at a constant stress intensity factor range ΔK was analyzed on the basis of a cumulative fatigue damage model. In this model, the points ahead of a crack tip are assumed to constitute a series of uniaxial fatigue specimens, and fatigue crack propagation is viewed as the successive fatigue failure of these specimens. Such steady state models have also been analyzed by Fleck and Anderson [2], Liu [3], Rice [4], and McClintock [5].

In this paper the same model is used to analyze the situation involving an increase in the stress intensity factor range from a lower level to a higher level. Such an analysis of the transient behavior of fatigue crack propagation appears to not yet have been carried out. Rather, the assumption is usually made that the crack propagation rate after a sudden increase in stress intensity factor range reaches the steady state rate corresponding to the higher cyclic load level in only a few cycles. The present analysis is an attempt to verify this assumption and to investigate the transient behavior as it relates to the low cycle fatigue properties of the material.

Analysis

The problem to be analyzed is shown in Fig. 1a. A crack which has been propagating at a constant stress intensity factor range ΔK_1, is suddenly subjected to a higher stress intensity factor range of ΔK_2. The X-Y coordinate system is stationary and has its origin at the crack tip location corresponding to the change in loading, while the x-y coordinate system is attached to the moving crack tip. At a later instant the crack length has increased by an amount a (Fig. 1b), so that a fatigue element at X is at a distance of x from the current crack tip location and

$X = x + a$. Initially, $a = 0$ and $X = x$. The initial damage of an element at X is given by

$$D_0(X) = \frac{1}{(da/dN)_1} \int_X^\infty \frac{1}{N_f(\Delta K_1, y)} \, dy \qquad (1)$$

where

$\left(\frac{da}{dN}\right)_1$ = steady state crack propagation rate at ΔK_1

$\frac{1}{N_f(\Delta K_1, y)}$ = damage experienced in one cycle by an element at a distance y from the crack tip at stress intensity factor range ΔK_1.

The damage of the same element after N cycles is

$$D(X, N) = D_0(X) + \int_0^N \frac{\partial D}{\partial N} \, dN \qquad (2)$$

where

$$\frac{\partial D}{\partial N} = \text{damage per cycle} = \frac{1}{N_f(\Delta K_2, x)} = \frac{1}{N_f(\Delta K_2, X - a)}$$

Also,

$$dN = \frac{da}{[da/dN(a)]}$$

where

$\frac{da}{dN}(a)$ = current crack propagation rate when the crack length increment is a.

If the element at X fails in N cycles, $D(X, N) = 1$, and Eq. 2 reduces to

$$D_0(X) + \int_0^X \frac{1}{N_f(\Delta K_2, X - a)} \frac{1}{[da/dN(a)]} \, da = 1$$

Using Eq. 1, the above can be written as

$$\int_0^X \Phi(a)\, L_2\,(X - a)\, da = \Phi_1 \int_0^X L_1\,(y)\, dy \tag{3}$$

where

$$\Phi(a) = \frac{1}{[da/dN\,(a)]}$$

$$\Phi_1 = \frac{1}{(da/dN)_1}$$

$$L_2\,(X - a) = \frac{1}{N_f\,(\Delta K_2,\ X - a)}$$

and

$$L_1\,(y) = \frac{1}{N_f\,(\Delta K_1,\ y)}$$

Equation 3 is a Volterra integral equation of the first kind and has to be solved for the unknown function $\Phi(a)$. The kernel is the damage per cycle function and can be related to the low cycle fatigue properties of the material as follows [1]:

$$L_i\,(X) = \frac{1}{N_f\,(\Delta K_i,\ X)} = \begin{cases} A_i\,(X + \alpha_i)^{-\beta} & \text{for } 0 \le X \le R_{pi} \\ 0 & \text{for } X > R_{pi} \end{cases} \qquad (i = 1, 2) \tag{4}$$

where R_{pi} is the reversed plastic zone size under the stress intensity factor range ΔK_i, and

$$A_i = 2\,[4(1 + n')\,\pi\,E\,\sigma_f'\,\epsilon_f']^{-\beta}\,\Delta K_i^{\,2\beta}$$

$$\beta = -\frac{1}{(b + c)}, \text{ usually varies between 1 and 2}$$

$$\alpha_i = \left(\frac{1}{2}\right) COD_i + \rho^* = \frac{\epsilon_y' \Delta K_i^2}{(\pi \sigma_y'^2)} + \rho^*$$

$$R_{pi} = \frac{\Delta K_i^2}{[4(1+n')\pi \sigma_y'^2]}$$

The steady state crack propagation rates are given by

$$\left(\frac{da}{dN}\right)_i = \frac{1}{\Phi_i} = \frac{A_i \alpha_i^{-\beta+1}}{\beta - 1} \left[1 - \left(\frac{1 + R_{pi}}{\alpha_i}\right)^{-\beta+1}\right], \quad i = 1, 2 \tag{5}$$

Taking Laplace transforms of Eq. 3, using Eq. 4 and solving for the Laplace transform of $\Phi(a)$, i.e. $\hat{\Phi}(p)$,

$$\hat{\Phi}(p) = \frac{\Phi_1 A_1}{A_2} \frac{e^{p(\alpha_1 - \alpha_2)}}{p} \frac{\Gamma(-\beta+1, \alpha_1 p) - \Gamma(-\beta+1, \alpha_1 p + R_{p1} p)}{\Gamma(-\beta+1, \alpha_2 p) - \Gamma(-\beta+1, \alpha_2 p + R_{p2} p)} \tag{6}$$

where

$\Gamma(-\beta+1, \alpha p) = \int_{\alpha p}^{\infty} e^{-y} y^{-\beta} dy$ is the incomplete Gamma function.

Approximation

It is difficult to either invert or obtain a useful asymptotic solution for Eq. 6. A reasonable asymptotic solution can be developed if the assumption is made that $R_{pi} \longrightarrow \infty$. For usual values of material properties the ratio R_{pi}/α_i is of the order of 50 to 100. If this assumption leads to a solution where the crack propagation rate is within a few percentage of the steady state value when the incremental crack length equals the reversed plastic zone size R_{p2}, it should give a reasonable approximation to the true solution.

Equations 5 and 6 can then be simplified to

$$\left(\frac{da}{dN}\right)_i = \frac{1}{\Phi_i}\frac{A_i\,\alpha_i^{-\beta+1}}{(\beta-1)} \tag{7}$$

$$\hat{\Phi}(p) = \frac{\Phi_1 A_1}{A_2}\,\frac{e^{p(\alpha_1-\alpha_2)}}{p}\,\frac{\Gamma(-\beta+1,\ \alpha_1 p)}{\Gamma(-\beta+1,\ \alpha_2 p)} \tag{8}$$

Though it is extremely difficult to invert Eq. 8, it is possible to obtain asymptotic solutions of $\Phi(a)$ for small and large values of the crack length extension a by constructing the asymptotic expansions of the right-hand side of Eq. 8 for large and small values of the parameter p respectively and then applying the theorems of Laplace transform theory [6]. Thus it can be shown that

$$\Phi(a) \backsim \Phi_1\Big[1 - \beta\left\{\frac{a}{\alpha_1} - \frac{a}{\alpha_2}\right\} + \frac{\beta(\beta+1)}{2}\left\{\left(\frac{a}{\alpha_1}\right)^2 - \left(\frac{a}{\alpha_2}\right)^2\right\} - \frac{\beta^2}{2}\left\{\frac{a^2}{\alpha_1\alpha_2} - \left(\frac{a}{\alpha_2}\right)^2\right\}\Big] + 0\,(a^3) \text{ as } a \longrightarrow 0 \tag{9}$$

and

$$\Phi(a) \backsim \Phi_2\left[1 + \left\{\left(\frac{\alpha_2}{a}\right)^{\beta-1} - \left(\frac{\alpha_1}{a}\right)^{\beta-1}\right\} + \frac{\beta-1}{2-\beta}\left\{\beta\left(\frac{\alpha_2}{a}\right)^{\beta} + (2-\beta)\left(\frac{\alpha_1}{a}\right)^{\beta} - \frac{\alpha_1\alpha_2}{a^2}\left[\left(\frac{\alpha_1}{a}\right)^{\beta-2} + \left(\frac{\alpha_2}{a}\right)^{\beta-2}\right]\right\}\right] + 0\,(a^{-\beta-1}) \text{ as } a \longrightarrow \infty \tag{10}$$

These two asymptotic solutions can, in fact, be extended to any number of terms desired.

RESULTS

Figures 2-4 show the variation of the average crack propagation rate (normalized with respect to the steady state crack propagation rate at ΔK_2) with the crack length increment a (as a fraction of the reversed plastic zone size at ΔK_2) for a low, medium, and high yield steels of comparable ductility and for two load ratios $\Delta K_2/\Delta K_1 = 2$ and 3. ΔK_1 was taken as 20 ksi$\sqrt{\text{in.}}$ in each case and the material properties used for computation are shown in Table 1. The upper curves correspond to the asymptotic solution for large values of crack length increment, a, taking two and three terms in Eq. 10, and the lower curves correspond to the asymptotic solution for small values of crack length increment, a, taking two and three terms in Eq. 9. The crack propagation rate rapidly approaches the steady state value in each case, reaching 99% of the full value for the low yield steel, 95% of the full rate for the medium yield steel and about 90% of the full rate for the high yield steel by the time the crack length increment equals the reversed plastic zone size R_{p2}. Hence, the assumption in the previous section ($R_p \longrightarrow \infty$) is reasonable.

The amount by which the crack grows initially in one cycle at the lower load level ΔK_1 is indicated in the lower part of the figures, and it is obvious that though the crack propagation rate increases rapidly with the crack length increment, it takes more than one cycle to increase the average crack propagation rate significantly. Notice that considering only three terms in the asymptotic solution for small crack length increment is sufficient for the low and medium yield steels, but not as good for the high yield steel.

Although theoretically the crack growth rate in every case reaches the steady state value exactly when the crack length increment equals the reversed plastic zone size R_{p2}, the rapidity with which the crack growth rate approaches the steady state value as a function of the crack length increment is controlled by the parameter β

(see Eq. 10). The value of β for the three steels vary from 1.33 for the high yield through 1.51 for the medium yield steel to 1.72 for the low yield steel. Consequently, the transient zone as a fraction of the reversed plastic zone size is largest for the high yield steel and smallest for the low yield steel. However, since the reversed plastic zone size for the high yield steel is much smaller than the low yield steel, the actual transient zone size for the low yield steel is larger than the high yield steel.

In order to determine how the crack length increment varies with each cycle, it is necessary to have the solution for crack propagation rate for all crack length increments. The present asymptotic solution provides this information only for small and large values of the crack length increment as shown typically by the solid lines in Fig. 5. Here the asymptotic solution for large crack length increment has been adjusted so that the crack propagation rate is 100% of the full steady state value when the crack length increment equals the reversed plastic zone size R_{p2}. These two asymptotic solutions are then joined by the dotted lines, and the area under the complete curve from 0 to a gives an estimate of the number of cycles required to cause the crack length increment a. Figures 6-8 shows the estimated variation in crack length increment with the number of cycles for the three steels. The dotted lines in these figures correspond to crack length increments if transient effects are ignored. Table 2 shows the estimated number of cycles and crack length increment necessary to achieve 90%, 95% and 100% of the full steady state crack propagation rate in each case. Notice that the crack length increment and number of cycles required to achieve any given fraction of the full steady state value vary inversely as the yield strengths of the steel. Also, for any given steel, the number of cycles required to attain a given percentage of the full steady state value is independent of the load ratio, whereas the crack length increment increases with the load ratio.

CONCLUSION

Upon increasing the stress intensity factor range and holding it steady, the crack propagation rate, according to the present model, does not immediately jump to the value corresponding to the higher stress intensity factor range.

There is a transient period, and the number of cycles and crack length increment necessary to achieve 90% or 95% of the full steady state crack growth rate decrease with increasing yield strength of the steel.

For any steel, the crack length increment required to attain 90% or 95% of the full steady state crack propagation rate increases with the load ratio $\Delta K_2/\Delta K_1$, whereas the number of cycles necessary to achieve the same crack growth rate is independent of the load ratio.

If the transient period is completely ignored and the crack is assumed to propagate at the steady state rate starting from the first cycle after the application of the higher load, the crack length, according to this analysis, is always overestimated by an amount of the order of the reversed plastic zone size at the higher load level.

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TABLE 1 MATERIAL PROPERTIES USED FOR COMPUTATION

Steel Strength	Modulus of Elasticity ksi	Yield* Strength ksi	Microstructure Size ρ^*, in.	Cyclic Strain Hardening Exponent n'	Fatigue Strength		Fatigue Ductility	
					Coefficient σ_f', ksi	Exponent b	Coefficient ϵ_f	Exponent c
High yield	28×10^3	180	10^{-5}	0.07	300	-0.05	1.0	-0.7
Medium yield	28×10^3	90	5×10^{-5}	0.10	200	-0.06	1.0	-0.6
Low yield	28×10^3	40	10^{-4}	0.15	100	-0.08	1.0	-0.5

*0.2% offset

TABLE 2 ESTIMATES OF CRACK GROWTH DURING TRANSIENT (ΔK_1 = 20 ksi$\sqrt{\text{in.}}$)

Steel Strength	ΔK_2 ksi$\sqrt{\text{in.}}$	Transient Crack Growth Rate as a % of Steady State Crack Growth Rate at ΔK_2	Crack Length Increment: a, in.	Crack Length Increment: % of Reversed Plastic Zone Size at ΔK_2	Estimated No. of Cycles after Increase in Loading
High yield	40	90%	6×10^{-4}	18.0%	22
		95%	12×10^{-4}	35.0%	40
		100%	34×10^{-4}	100.0%	104
	60	90%	18×10^{-4}	24.0%	30
		95%	32×10^{-4}	41.0%	49
		100%	77×10^{-4}	100.0%	106
Medium yield	40	90%	23×10^{-4}	15.0%	140
		95%	45×10^{-4}	30.0%	248
		100%	156×10^{-4}	100.0%	767
	60	90%	57×10^{-4}	16.0%	158
		95%	124×10^{-4}	35.0%	297
		100%	352×10^{-4}	100.0%	749
Low yield	40	90%	60×10^{-4}	9.0%	600
		95%	114×10^{-4}	17.0%	980
		100%	690×10^{-4}	100.0%	4910
	60	90%	153×10^{-4}	10.0%	650
		95%	286×10^{-4}	18.0%	1060
		100%	1560×10^{-4}	100.0%	4610

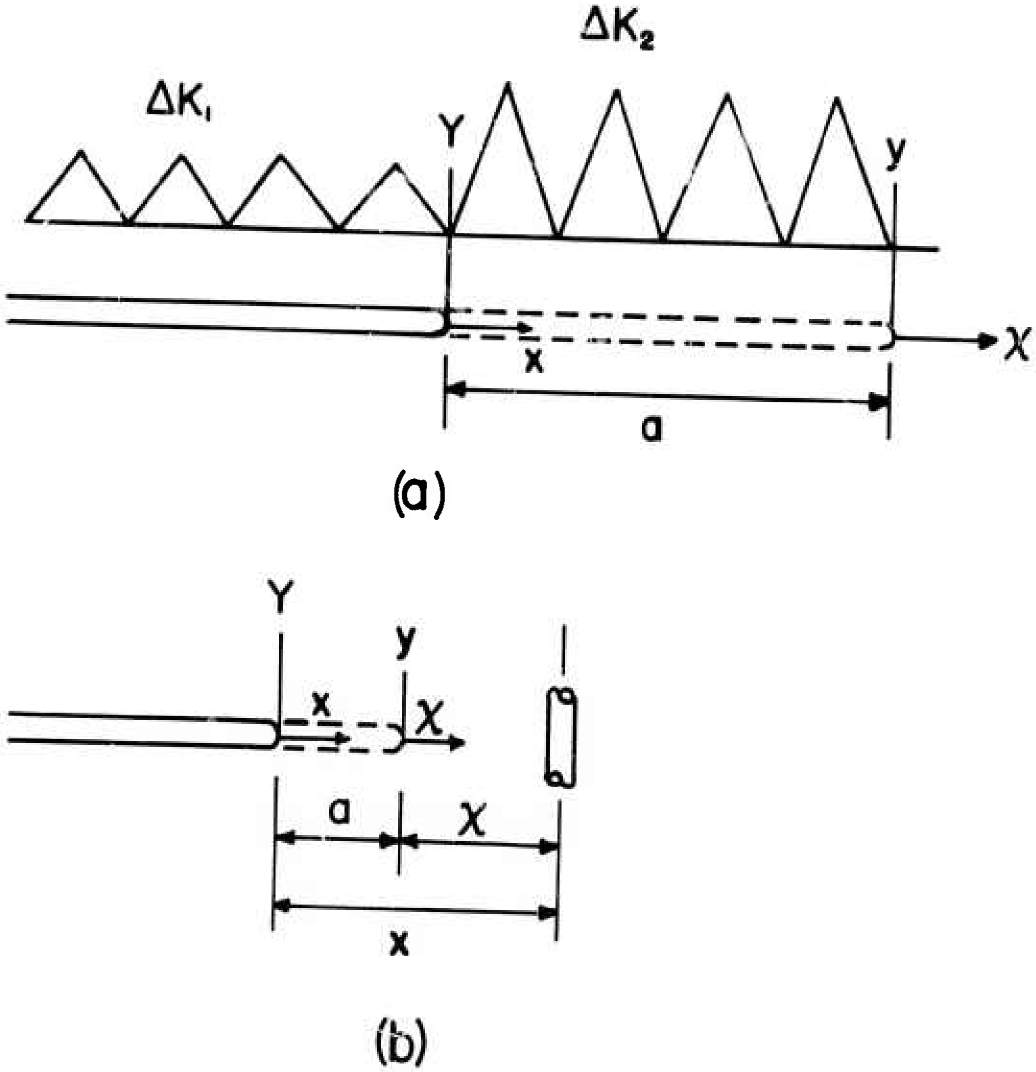

Fig. 1 Coordinates and Geometry Associated with a Mode I Crack

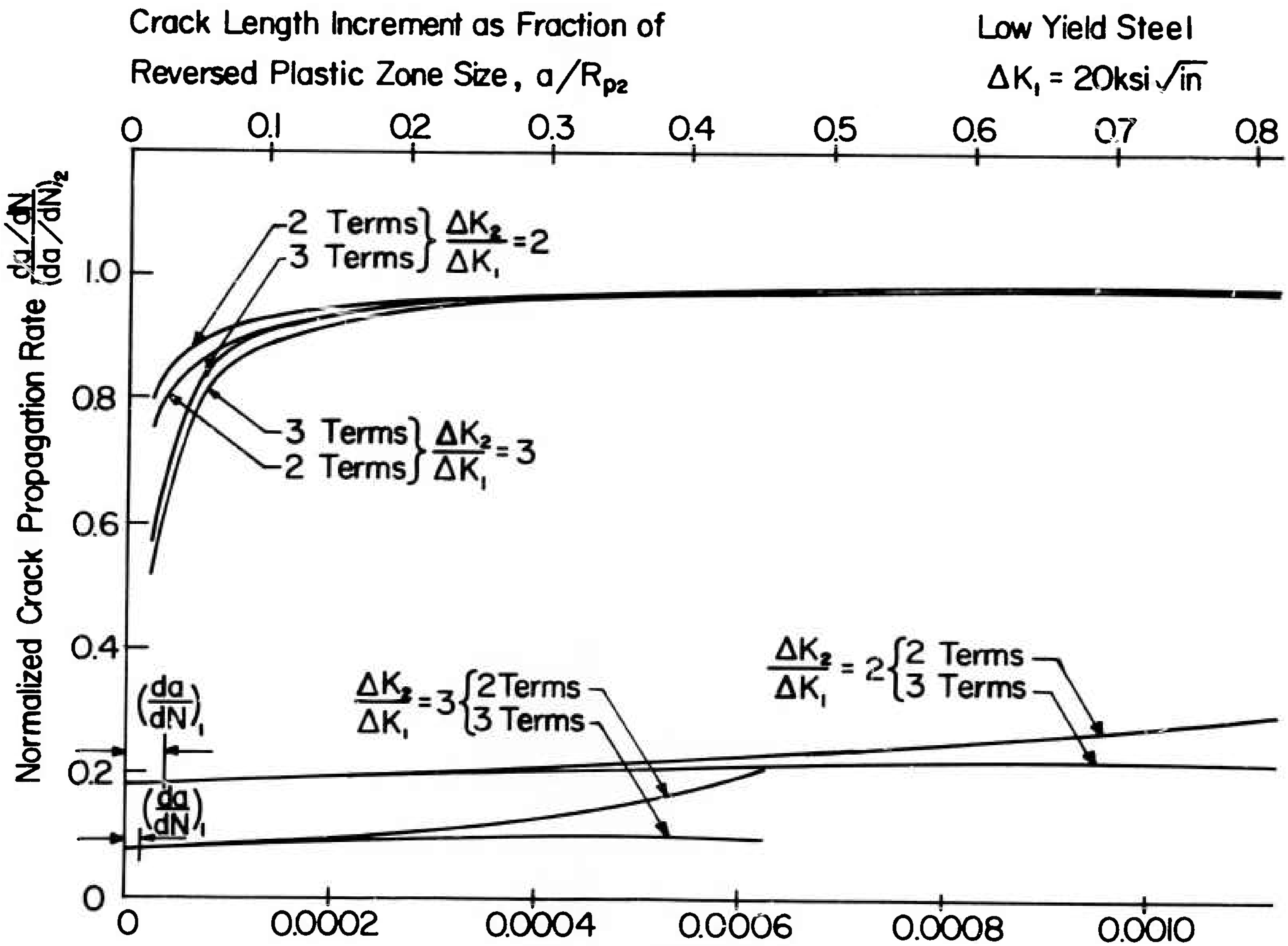

Fig. 2 Crack Length Increment as Fraction of Reversed Plastic Zone Size at ΔK_2, a/R_{p2}

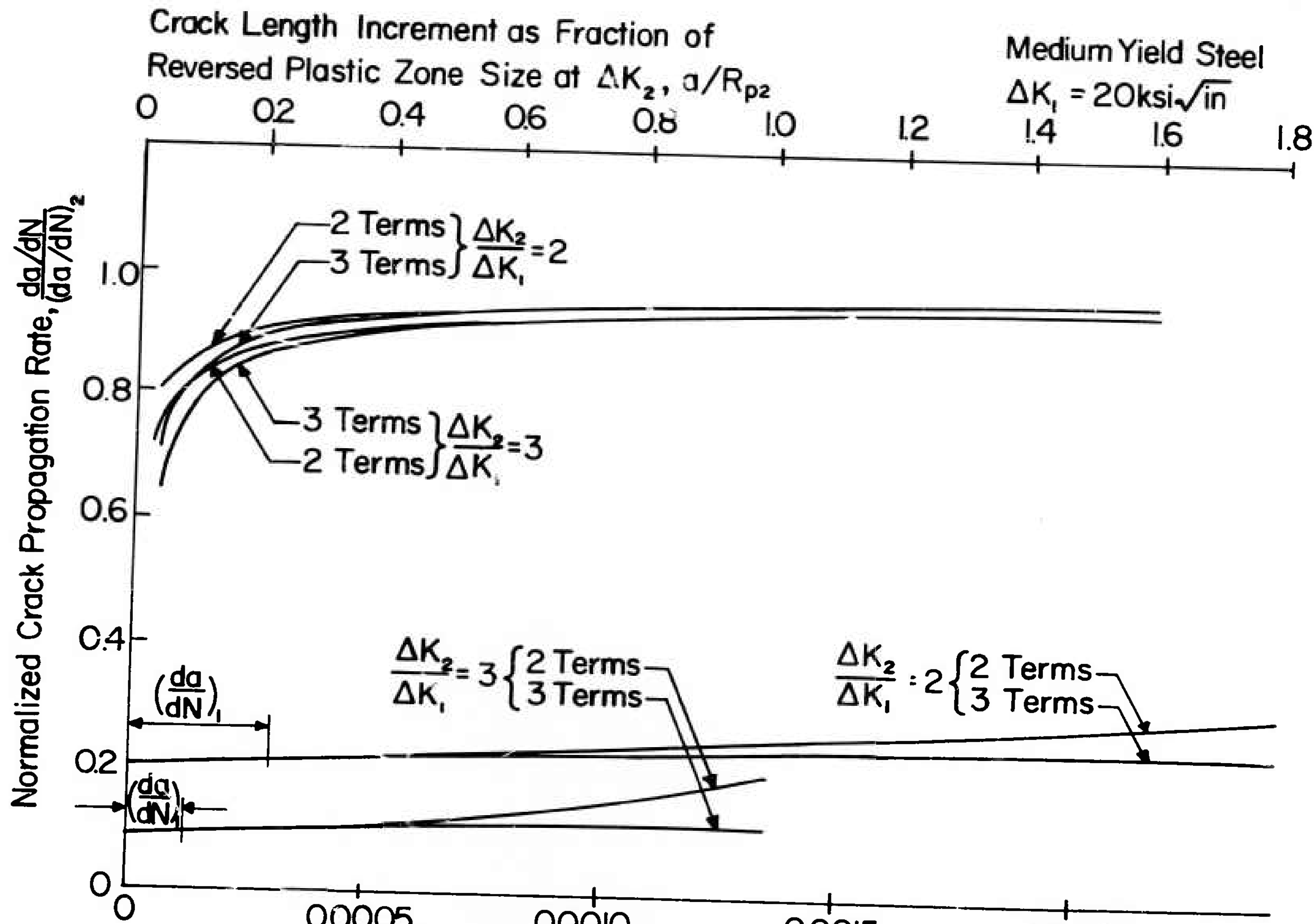

Fig.3 Crack Length Increment as Fraction of Reversed Plastic Zone Size at ΔK_2, a/R_{p2}

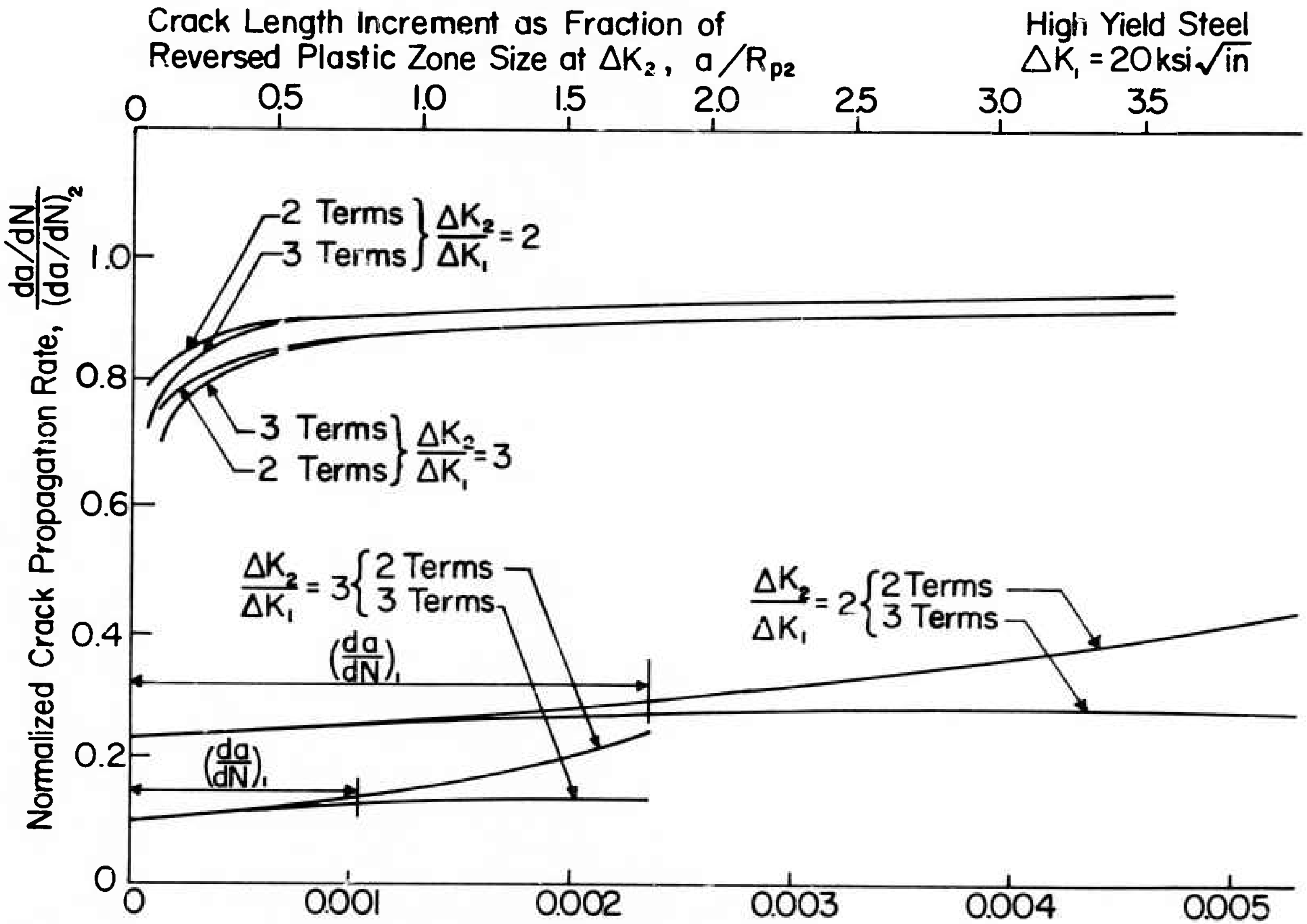

Fig.4 Crack Length Increment as Fraction of Reversed Plastic Zone Size at ΔK_2, a/R_{p2}

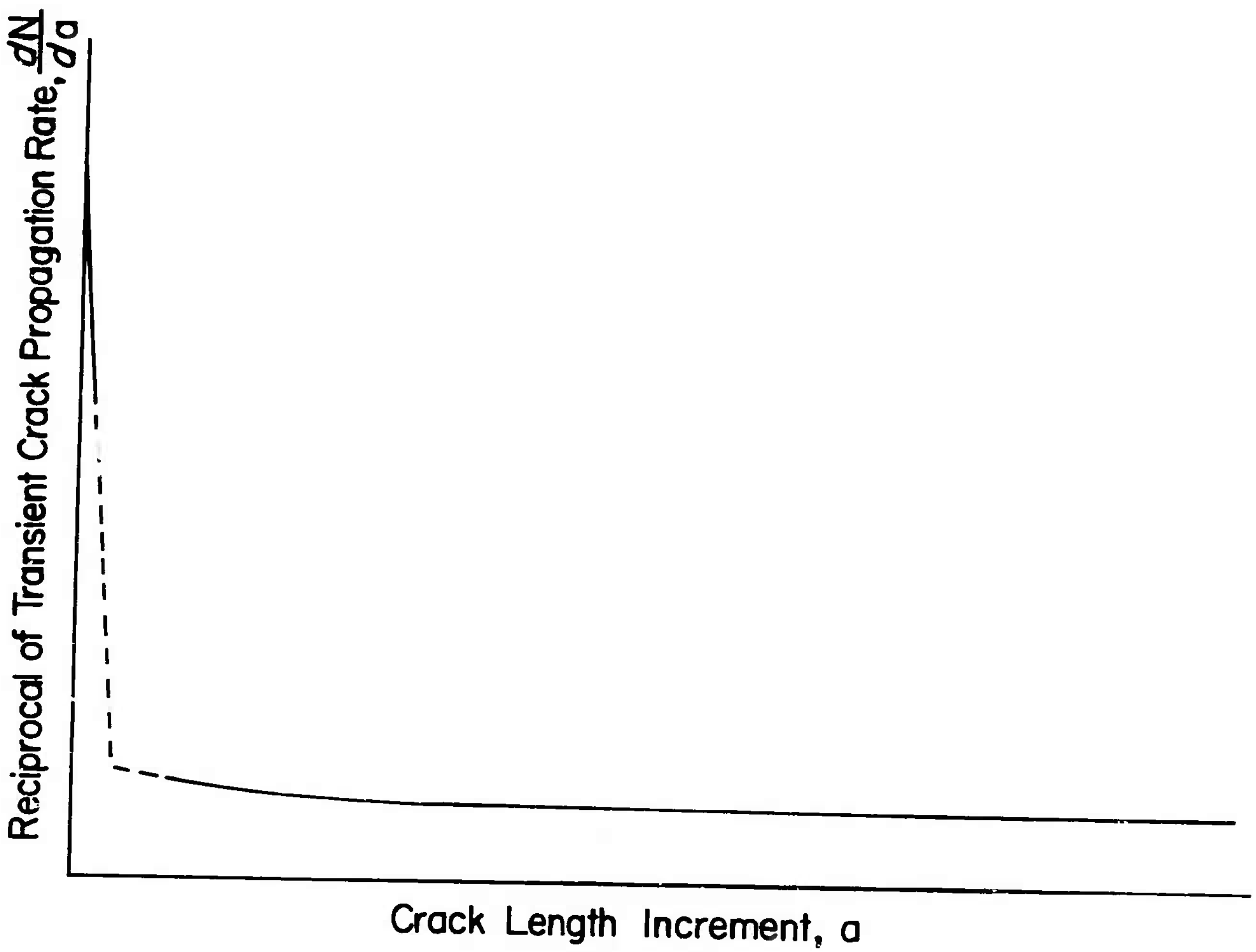

Fig.5 Method of Estimating Crack Propagation Rate for Intermediate Values of Crack Length Increment

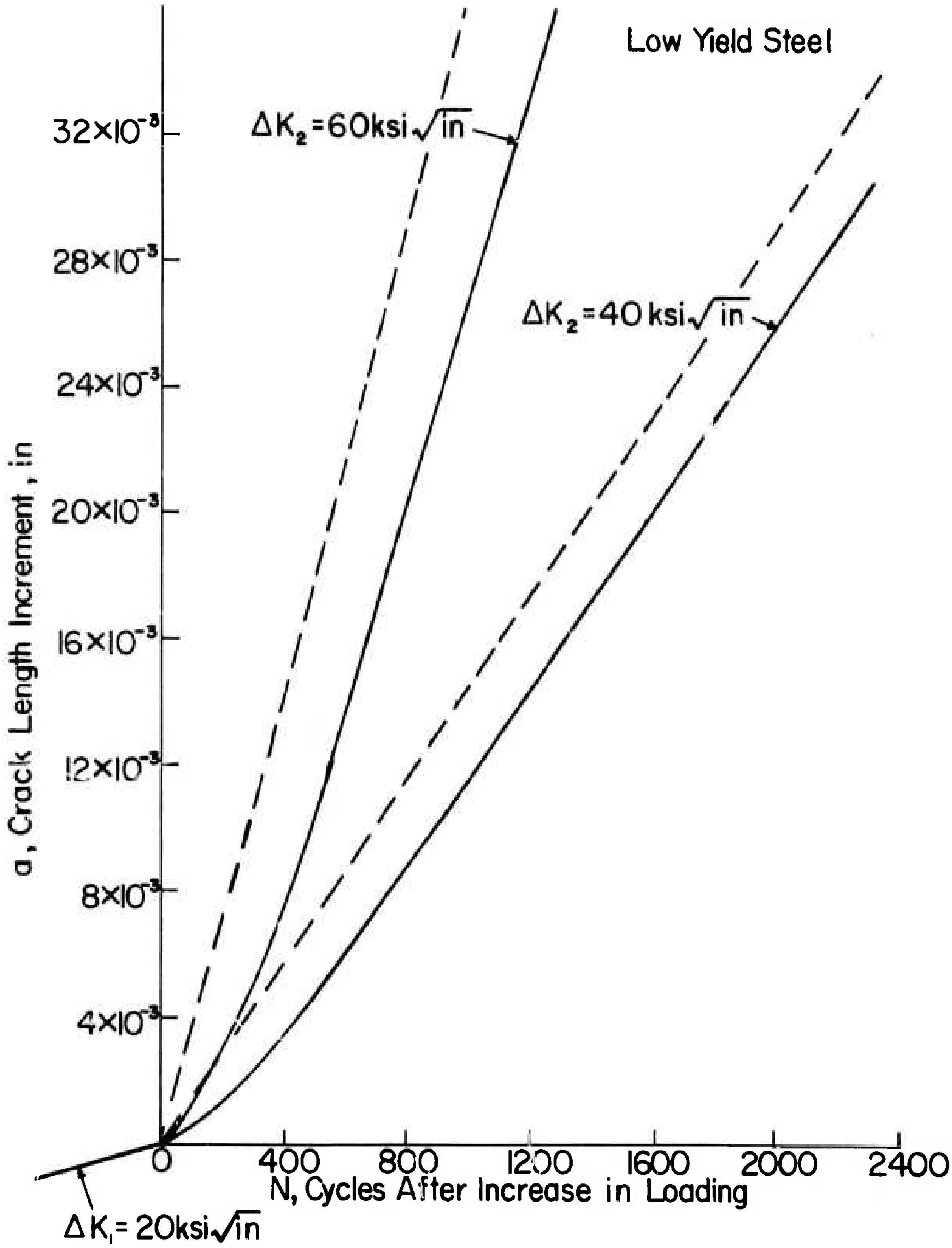

Fig.6 Estimate of Crack Length Increment versus Cycles for a Low Yield Steel

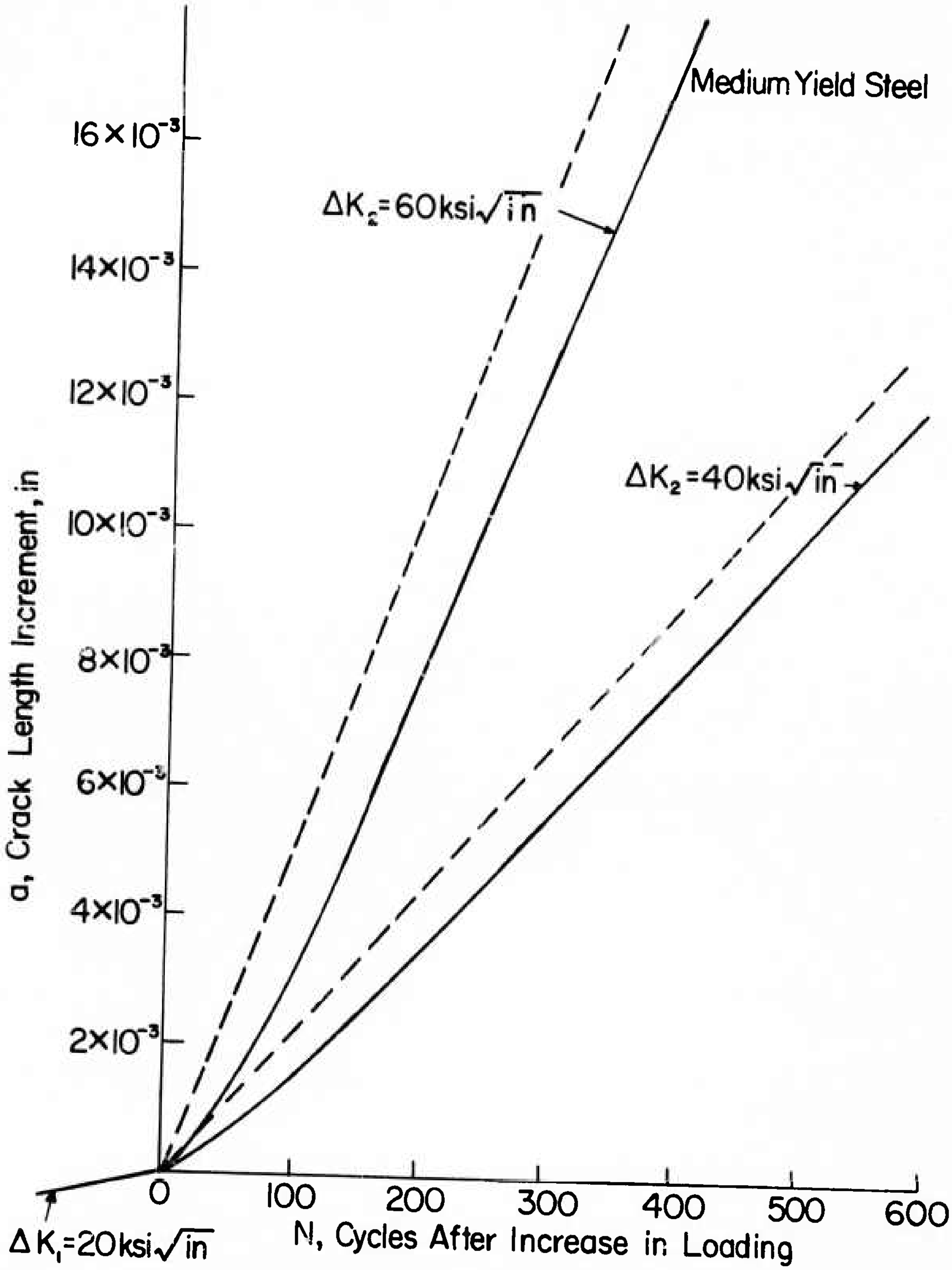

Fig. 7 Estimate of Crack Length Increment versus Cycle for a Medium Yield Steel

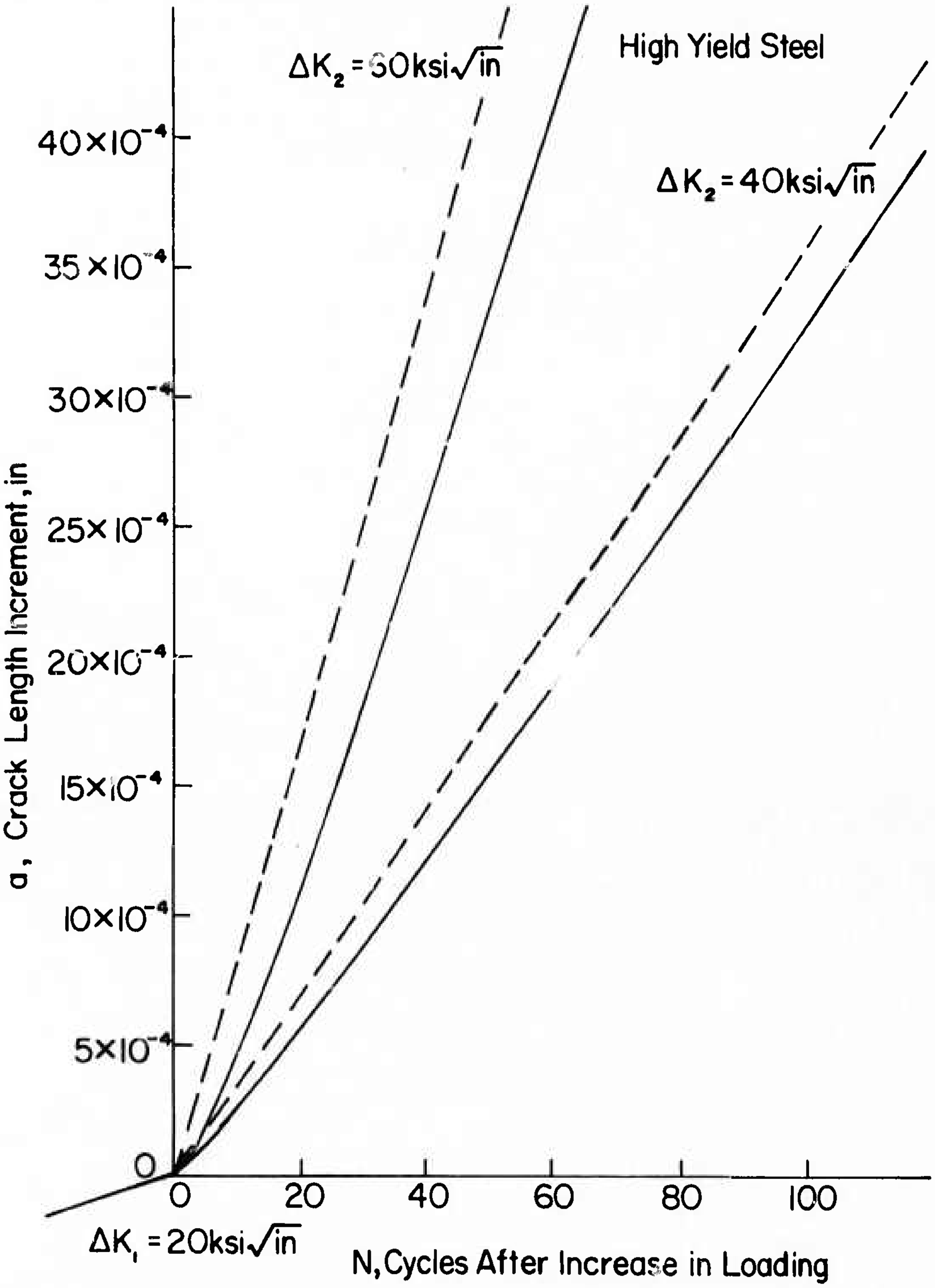

Fig. 8 Estimate of Crack Length Increment versus Cycle for a High Yield Steel